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# A novel method for controller design in engineering education

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ABSTRACT: This study proposes a novel method, using a baseline sliding mode controller (BSMC) and a discrete linear quadratic regulator (LQR) based on the fireworks algorithm (FWA), for controller design to optimise controller parameters. The dominant performance of the proposed method is verified on a nonlinear inverted pendulum system. The simulation process is carried out using MATLAB/Simulink. The results are compared with a published method that designs a hybrid control of a proportional-integral-derivative controller (PID) and LQR. The simulation results show a better performance of the proposed controller.

## INTRODUCTION

Engineering education places a heavy priority on laboratory experience. Computer-based simulation is being widely used for the purposes of engineering education [1]. A paper by Georgiev et al presents experiences in building simulation laboratories and provides a discussion of important and relevant issues with regard to the pedagogy, software and equipment utilised [2].

The inverted pendulum system is one of the more popular benchmarks in automatic control that is used to verify the performance of control techniques. The aims of a designing controller for the inverted pendulum system are not only to guarantee the cart can move to a desired position by changing the external force, but also to stabilise the pendulum balanced in the upright position. Because of high nonlinearity, controllers that are designed for the inverted pendulum system should not be pure linear controllers, such as a proportional-integral-derivative controller (PID) and a linear quadratic regulator (LQR) [3]. In fact, there are many pure nonlinear controllers that have had their performance verified, such as model predictive control [4], sliding mode control [5][6], fuzzy logic control [7][8], fuzzy-neural control [9], and so on. In recent years, many high-performance hybrid controls have been designed, such as the PID hybrid control design and the linear quadratic controller (LQR) [10].

The authors of this article propose a hybrid control design that combines a baseline sliding mode controller (BSMC) [11] and a discrete LQR [6]. However, in order to enhance performance of the system, some intelligence optimisation algorithms are used, such as particle swarm optimisation (PSO) [12], genetic algorithm (GA) [13] or the bee colony algorithm [14]. Besides these heuristic algorithms, an evolutional algorithm, such as the fireworks algorithm (FWA) has been applied widely in recent times [15][16]. The advantages of FWA are known as higher optimisation accuracy and faster convergence speed as compared with PSO [15].

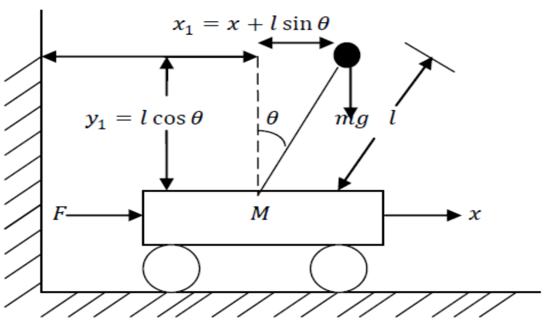



Figure 1: The inverted pendulum system.

In this article, the DLQR is used to stabilise the pendulum balanced in the upright position, while BSMC is employed to enhance the desired position in tracking performance. The parameters of BSMC and DLQR controllers are optimised by the fireworks algorithm. This article is organised as follows. The second section describes the methodology for this system. The third section shows the simulation and results, whereas the conclusions are given in the last section.

## METHODOLOGY

### Modelling Inverted Pendulum System

Figure 1 shows the inverted pendulum system. The system contains two parts: cart and pendulum (rod), where $\theta$ is the angle of the inverted pendulum from the vertical axis, x is the displacement of the cart, and $F$ is a driving force applied to the system. $M$ is the mass of the cart; m is the ball point mass as the upper end of the inverted pendulum (mass of the rod is negligible); $l$ is the length of the pendulum rod. Assume that the inertia moment of pendulum and frictional force are negligible.

By applying the Euler-Lagrange law and conservation law, the dynamics equations of the system are shown as [5]:

$$\begin{cases} \ddot{x} = \dfrac{F + ml\sin(\theta)\dot{\theta}^2 - mg\sin\theta\cos\theta}{M + m - m\cos^2\theta} \\ \ddot{\theta} = \dfrac{F\cos\theta - (M+m)g\sin\theta + ml(\sin\theta + \cos\theta)\dot{\theta}^2}{ml\cos^2\theta - (M+m)l} \end{cases} \tag{1}$$

The state variables are defined as position and velocity of the cart, angle and angular velocity of the pendulum, respectively:

$$x_1 = x; \quad x_2 = \dot{x}; \quad x_3 = \theta; \quad x_4 = \dot{\theta}. \tag{2}$$

Then, the above dynamic equations can be represented in the form of the state space equations as below:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{F + ml\sin(\theta)\dot{\theta}^2 - mg\sin\theta\cos\theta}{M + m - m\cos^2\theta} \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{F\cos\theta - (M+m)g\sin\theta + ml(\sin\theta + \cos\theta)\dot{\theta}^2}{ml\cos^2\theta - (M+m)l} \end{cases} . \tag{3}$$

The output equation can be written as:

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix}. \tag{4}$$

### Discrete Linear Quadratic Regulator (DLQR)

From the dynamic Equations (3) and the output Equation (4), the system is linearised in the form of the following state space equations:

$$x = Ax + Bu \tag{5}$$

Now, the system is discretised at the sampling time $\tau$ that is shown as the followings:

$$x(k+1) = \Phi x(k) + \Gamma u(k) \tag{6}$$

The quadratic equation is shown as:

$$J = \int_0^\infty [\mathrm{X}^T Q \mathrm{X} + u^T Ru]dt \tag{7}$$

The aim of designing the LQR controller is to design the state feedback matrix $F$ satisfying $u = Fx$ that can stabilise the system and minimise the cost function $J$, where $F$ is given by $F = R^{-1}B^T P$ and $P$ is determined by solving the continuous time algebraic Riccati equation:

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \tag{8}$$

where $Q$ and $R$ are weighted matrices that can be pre-defined.

In order to simplify the calculation and realisation, the $Q$ matrix can be defined as the symmetric positive semi-definite matrix whose parameters are optimised by FWA, while $R$ is a symmetric positive definite matrix, and is set to 1.

$$Q = diag(q_1, q_2, q_3, q_4) \text{ and } R = 1 \tag{9}$$

Baseline Sliding Mode Control (BSMC)

There are two key steps in designing a typical sliding mode control; they are to design control law $u_t$ and sliding surface function $s_t$. The control law of BSMC is the same as a standard SMC as below:

$$u_t = -Msign(s_t). \tag{10}$$

where $M$ is the gain.

The sliding surface function of BSMC is formed by cascading two controllers; namely, PID and PI [11].

$$s_t = (\lambda \times e + (\frac{\lambda}{2})^2 \sum e) \times (\lambda \times e + \dot{e} + (\frac{\lambda}{2})^2 \sum e) \tag{11}$$

where $\lambda$ is the gain, $e$ is the error signal between the reference variable and the position output variable. Both values $M$ and $\lambda$ are also optimised by the fireworks algorithm.

Fireworks Algorithm (FWA)

Figure 2 shows the framework of FWA. Similar to the explosion phenomenon of a real fireworks, a shower of sparks will enter the local space around a firework when it is set off. In this way, the explosion process of a firework can be viewed as a search in the local space around a specific point where the firework is set off through the sparks generated in the explosion. At the beginning of FWA, there are $n$ fireworks, which are set off at $n$ given locations. Then, after an explosion, the locations of the sparks are evaluated. When the optimal location is found, the algorithm stops. Otherwise, $n$ other locations are selected from the current sparks and the current fireworks for the next generation of explosions.

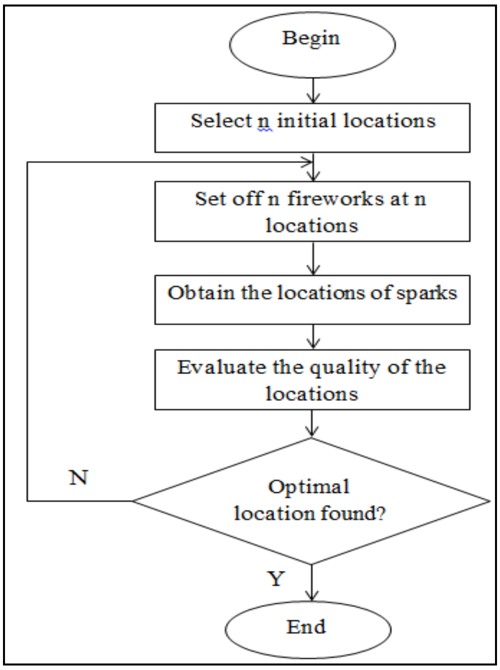



Figure 2: The framework of FWA program.

Cost Function

The cost function can be formulated from one or many different performance criteria. In this article, there are three typical performance criteria, given as follows.

Integral of square time multiplied by square error (ISTSE) is as follows:

$$ISTSE = \int t^2 e^2(t) dt \tag{12}$$

where *t* denotes the current evaluation time, *e(t)* is error value between set point and current output.

Mean square error (MSE) is as follows:

$$MSE = \frac{1}{n} \sum_{i=1}^{n} e^2(t) \tag{13}$$

where *n* is the length of simulation time.

The ISTSE, ITSE performance criteria can make the system response to overcome the disadvantages of integral of absolute value of error (IAE) and integral square error (ISE). However, it does not mean minimising all the performance parameters of system response, such as the percent of overshoot (P.O.), settling time ($T_s$), rising time ($T_r$) or steady state ($E_{ss}$) at the same time [17].

In multi-objective optimisation problems, the Pareto method, which optimises many different objectives at the same time is a very popular. Nevertheless, when the number of objective functions increases, using this method becomes a hard task because of high complexity. In this article, the performance criteria are combined in a single weighted sum objective function that is defined as the following function:

$$J = \sum_{i=1}^{n} w_i f_i(k) \tag{14}$$

where $f_i(k)$ is the performance criterion, *n* is the number of performance criteria, and $w_i$ is the weighted value of each performance criterion, such that:

$$\sum_{i=1}^{j} w_i = 1 \tag{15}$$

SIMULATION AND RESULTS

The Figure 3 and Figure 4 show the simulation model of the optimisation process and the hybrid control configuration, respectively.

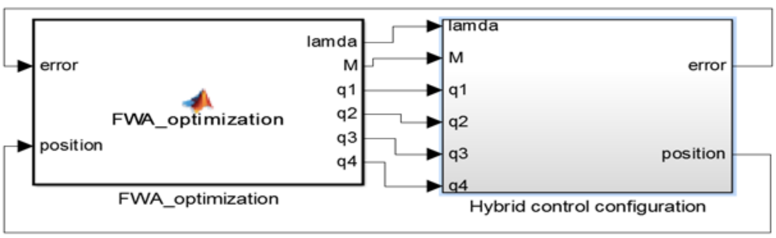



Figure 3: Simulation model for optimisation process.

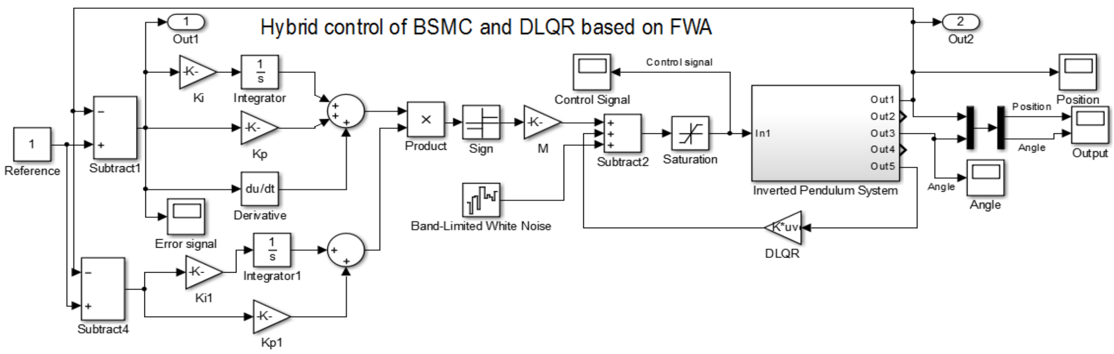



Figure 4: Simulink model of hybrid control configuration.

The simulation parameters of an inverted pendulum system are set as those in [5][10], with: mass of the cart 2.4 kg, mass of the pendulum 0.23 kg, length of pendulum 0.36 m, gravity 9.8 m/s$^2$ and driving force in the range of [-20 20] N.

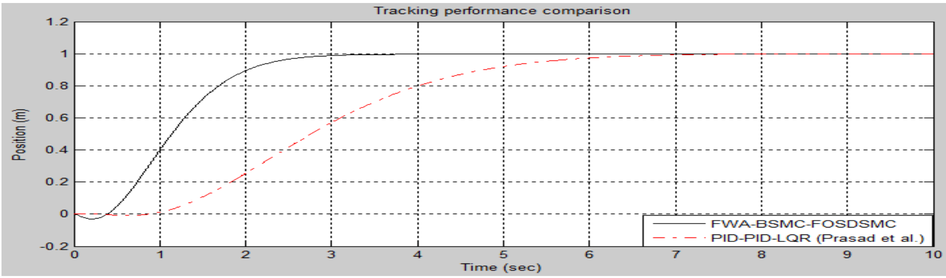
In Figure 3, the *FWA_optimisation* block is a function that is employed to optimise six variables by using FWA, including two parameters of the BSMC controller and four parameters of Q matrix in DLQR controller. The proposed cost function of optimisation process is formulated by ISTSE, MSE, and the percentage of overshoot parameter (P.O.) is as follows:

$$J = \alpha_1 * ISTSE + \alpha_2 * MSE + \alpha_3 * P.O. \tag{16}$$

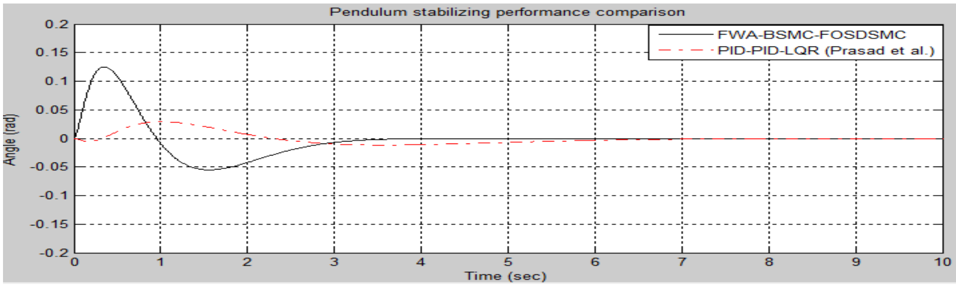
The combination of these performance criteria, especially P.O. can guarantee that the controller can result in a system response with the smallest value of overshoot and the shortest possible settling time.

The parameters of the cost function are set as: $\alpha_1 = 0.25; \alpha_2 = 0.1; \alpha_3 = 0.6$. The typical parameters of the fireworks algorithm are set as: six optimised variables, including $\lambda$ in range of [-200 200] and the gain *M* in the range of [-20 20], all four variables of Q matrix ($q_1$, $q_2$, $q_3$, $q_4$) are in the range of [0.1 1000]; the number of fireworks *n* = 6; the value of the total number of sparks *m* = 64; the maximum explosion amplitude *A* = 2; maximum iterations 400; maximum evaluation 50000; *a* = 0.04, *b* = 0.8; the DLQR controller parameters are designed at sampling time 0.01s. A band limited white noise block in MATLAB/Simulink is chosen as the disturbance input whose parameters are set as: power is 0.001, sample time is 0.01 and seed is 23341.

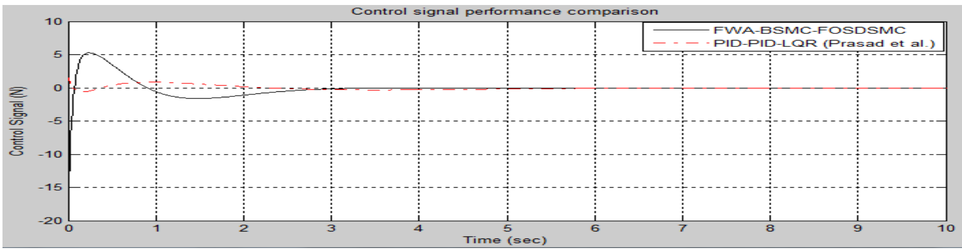
The dominant performance of the proposed controllers is verified by comparing with a published controller that uses two PID controllers in parallel for the outer loop and a state feedback controller based on linear quadratic regulator for the inner loop. The position tracking, angle stabilising performance and control force of both controllers are shown in Figure 5.




a)



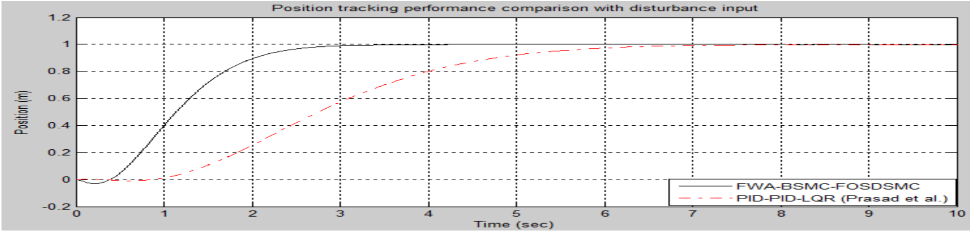

b)



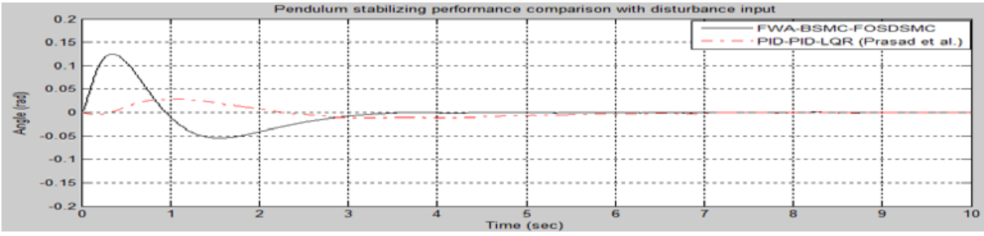

c)

Figure 5: Response of a) position; b) angle; and c) control force with no disturbance input between the proposed controller and Prasad et al's controller.

In addition, the proposed controller is verified in the case of disturbance input, and is shown in Figure 6. The optimised controller parameters and the performance in transient response of cart position for both controllers are listed in Table 1 and Table 2, respectively.




a)

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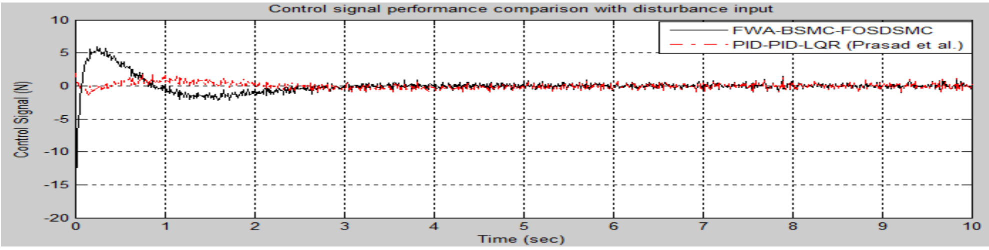


c)

Figure 6: Response of a) position; b) angle; and c) control force with disturbance input between the proposed controller and Prasad et al's controller.

Table 1: Optimised controller parameters.

| Controllers | Proposed controller parameters | | | | | |
|---|---|---|---|---|---|---|
| | $\lambda$ | $M$ | $q_1$ | $q_2$ | $q_3$ | $q_4$ |
| FWA-BSMC-DLQR | 1.195 | 19.03 | 488.48 | 168.51 | 9.362 | 386.5 |

Table 2: The target tracking performance comparison.

| Controllers | Performance criteria | | |
|---|---|---|---|
| | P.O. (%) | $T_s$ (s) | $T_r$ (s) |
| FWA-BSMC-DLQR (Proposal) | 0 | 2.56 | 1.44 |
| PID-PID-LQR (Prasad et al's) | 0 | 5.9 | 3.30 |

For target tracking control, the proposed controller results in a system response with much smaller values in both settling time and rising time as compared with the Prasad et al controller. With such a faster response in the position tracking, consequently, the response in the pendulum has more oscillations, as well as slightly higher value of overshoot. However, the value of overshoot is small enough to guarantee making the pendulum stable in the upright position in as short a time and as fast as possible. In the case with disturbance input, the proposed controller still not only guarantees stabilising the inverted pendulum system, but also has the better response of position and angle than those of Prasad et al's controller. The input control signal of the proposed controller is also small enough in a suitable range.

## CONCLUSIONS

In engineering education, a novel methodology to optimise all parameters of controllers in the hybrid control design of baseline sliding mode controller and discrete LQR is proposed, based on the fireworks algorithm. The proposed controller not only controls the cart to the desired position quickly and accurately, but also guarantees stabilising pendulum balanced in the upright position as fast as possible. The simulation results show a better performance of the proposed controller than that in Prasad et al's [10].

REFERENCES


1. Shyr, W-J., Enhancement of PLC programming learning based on a virtual laboratory. *World Trans. on Engng. and Technol. Educ.*, 8, **2**, 196-202 (2010).
2. Georgiev, S., Roth, H., Stefanova, S., Georgiev, T., Stoyanov, E. and Rosch, O., How and why to build and use virtual laboratories. *World Trans. on Engng. and Technol. Educ.*, 1, **2**, 191-195 (2002).
3. Sharif, B.A. and Ucar, A., State feedback and LQR controllers for an inverted pendulum system. *Proc. Inter. Conf. on Technol. Advanced in Electrical, Electronics and Computer Engng. (TAEECE)*, Konya, 298-303 (2013).
4. Cheng, C., Zhao, Z. and Li, H., MPC controller performance evaluation and tuning of single inverted pendulum device. *J. of Computer*, 8, **6**, 1560-1570 (2012).
5. Reddy, N.P.K., Kumar, M.S and Rao, D.S., Control of nonlinear inverted pendulum system using PID and fast output sampling based discrete sliding mode controller. *Inter. J. of Engng. Research & Technol.*, 3, **10**, 1000-1006 (2014).
6. Ngadengon, R., Sam, Y.M., Osman, J.H.S. and Ghazali, R., Controller design for inverted pendulum system using discrete sliding mode control. *Proc. Inter. Conf. on Instrumentation Control and Automation*, Bandung, Indonesia, 130-133 (2011).
7. Jain, A., Tayal, D. and Sehgal, N., Control of nonlinear inverted pendulum using fuzzy logic controller. *Inter. J. of Computer Applications*, 69, **27**, 7-11 (2013).
8. Cuevas, P.T., Luna, A.H. and Sanchez, J.F.H., Stability of fuzzy and LQR controllers applied to an inverted pendulum system. *Proc. 2015 IEEE Int. Autumn Meeting on Power, Electronics and Computing (ROPEC)*, Ixtapa, Zihuatanejo, Mexico, 1-6 (2015).
9. Wai, R.J. and Chen, P.C., Robust neural-fuzzy network control for robot manipulator including actuator dynamics. *IEEE Trans. on Industrial Electronics*, 53, **4**, 1328-1349 (2006).
10. Prasad, L.B., Tyagi, B. and Gupta, H.O., Optimal control of nonlinear inverted pendulum system using PID controller and LQR: Performance analysis without and with disturbance input. *Inter. J. of Automation and Computing*, 11, **6**, 661-670 (2014).
11. Piltan, F., Mehrara, S., Bayat, R. and Rahmdel, S., Design new control methodology of industrial robot manipulator: sliding mode baseline methodology. *Inter. J. of Hybrid Infor. Technol.*, 5, **4**, 41-54 (2012).
12. Hanafy, Th.O.S., Stabilization of inverted pendulum system using particle swarm optimization. *Proc. 8th Inter. Conf. of Informatics and Systems*, Cairo, 207-210 (2012).
13. Moghaddas, M., Dastranj, R. and Changizi, N., Design of optimal PID controller for inverted pendulum using genetic algorithm. *Inter. J. of Innovation, Manage. and Technol.*, 3, **4**, 440-442 (2012).
14. Wang, H.Q., Zhou, H.Q., Wang, D.Y. and Wen, S.J., Optimization of LQR controller for inverted pendulum system with artificial bee colony algorithm. *Proc. Inter. Conf. on Advanced Mechatronic Systems*, Luoyang, China, 158-162 (2013).
15. Tan, Y. and Zhu, Y.C., *Fireworks Algorithm for Optimization*. In: Advances in Swarm Intelligence. Beijing, China: Springer Verlag, 355-364 (2010).
16. Zheng, S., Janecek, A., Li, J. and Tan, Y., Dynamic search in fireworks algorithm. *Proc. of IEEE Congress on Evolutionary Computation (CEC)*, Beijing, 3222-3229 (2014).
17. Sahib, M.A. and Ahmed, B.S., A new multi-objective performance criterion used in PID tuning optimization algorithms. *J. of Advanced Research*, 1-9 (2015).